



(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9618

Roll No.

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B.Tech

**(SEM III) ODD SEMESTER THEORY EXAMINATION 2009-10
MATHEMATICS-III (SCIENCE BASED)**

Time : 3 Hours]

[Total Marks : 100

Note : Attempt all the questions.**1** Attempt any **four** parts of the following : **5×4=20**

(a) Define a harmonic function and conjugate harmonic function. Find the harmonic conjugate function of the function $U(x, y) = 2x(1 - y)$.

(b) Evaluate $\int_C (12z^2 - 4iz)$ along the curve C joining

the points (1, 1) and (2, 3).

(c) State the Cauchy's integral formula. Show that

$$\int_C \frac{e^{zt}}{z^2 + 1} dz = \sin t$$

if $t > 0$ and C is the circle $|z| = 3$.

(d) Define the Laurent series expansion of a function.

Expand $f(z) = e^{z/z-2}$ in a Laurent series about the point $z = 2$.



(e) Using Residue theorem, evaluate

$$\frac{1}{2\pi i} \int_C \frac{e^{zt} dz}{z^2(z^2 + 2z + 2)}$$

where C is the circle $|z| = 3$.

(f) Show that
$$\int_0^{2\pi} \frac{\cos 3\theta d\theta}{5 - 4 \cos \theta} = \frac{\pi}{12}$$

2 Attempt any **two** parts of the following : $10 \times 2 = 20$

- (a) The first four moments of a distribution about the value '0' are -0.20, 1.76, -2.36 and 10.88. Find the moments about the mean and measure the kurtosis.
- (b) Fit a second degree parabola to the following data

x	1	2	3	4	5	6	7	8	9	10
y	124	129	140	159	228	289	315	302	263	210

- (c) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible. Variance of $x = 9$.

Regression lines

$$8x - 10y + 66 = 0$$

$$40x - 18y - 214 = 0$$

What were

- (i) The mean value of x and y
- (ii) The standard deviation of y and
- (iii) The coefficient of correlation between x and y .



3 Attempt any **two** parts of the following : $10 \times 2 = 20$

- (a) To test the effectiveness of inoculation against cholera, the following table was obtained.

	<i>Attacked</i>	<i>Not attacked</i>	<i>Total</i>
<i>Inoculated</i>	30	160	190
<i>Not inoculated</i>	140	460	600
<i>Total</i>	170	620	790

(The figures represent the number of persons).

Use χ^2 -test to Defend or refute the statement. The inoculation prevents attack from cholera.

- (b) If there are 3 misprints in a book of 1000 pages, find the probability that a given page will contain (i) no misprint (ii) more than 2 misprints.

- (c) Show that the mean deviation from the mean of the normal distribution is about $\frac{4}{5}$ of its standard deviation.

4 Attempt any **two** parts of the following : $10 \times 2 = 20$

- (a) Derive the Newton-Raphson formula for finding a root of a non-linear equation. Find a root of

$$f(x) = x^3 + 2x^2 + 10x - 20 = 0$$

up to 10 iterations.

- (b) Define the shift operator, forward and backward difference operators, the central difference operator and the average operators. Establish :

$$(i) \quad \left(E^{\frac{1}{2}} + E^{-\frac{1}{2}} \right) = 2 \left(1 + \frac{1}{2} \Delta \right) (1 + \Delta)^{1/2}$$

$$(ii) \quad \Delta(1 + \Delta)^{-1/2} = \nabla(1 - \nabla)^{-1/2}$$



$$(iii) \mu = \sqrt{\left(1 + \frac{1}{4}\delta^2\right)}$$

where all the above notations have usual meanings.

- (c) Develop the divided-difference table from the data given below and obtain the interpolation polynomial $f(x)$:

x	1	3	5	7	11
$f(x)$	5	11	17	23	29

also, find the value of $f(19.5)$.

- 5 Attempt any **two** parts of the following : 10×2=20

- (a) Describe a method for solving a system of linear equations. Solve the following system of linear equations using Gauss-Seidel method :

$$23x_1 + 13x_2 + 3x_3 = 29$$

$$5x_1 + 23x_2 + 7x_3 = 37$$

$$11x_1 + x_2 + 23x_3 = 43$$

- (b) Derive Simpson's $\frac{3}{8}$ -formula for numerical

integration. Using this rule evaluate $\int_0^1 \frac{dx}{x^3 + x + 1}$.

Choose with steplength 0.25.

- (c) Solve the following initial value problem

$$\frac{dy}{dx} = -2xy^2, y(0) = 1$$

with $h = 0.2$ on interval $[0, 0.6]$ using fourth order Runge-Kutta method. Compare with the exact solution.

